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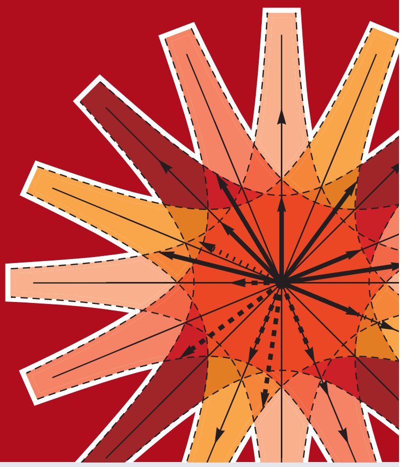
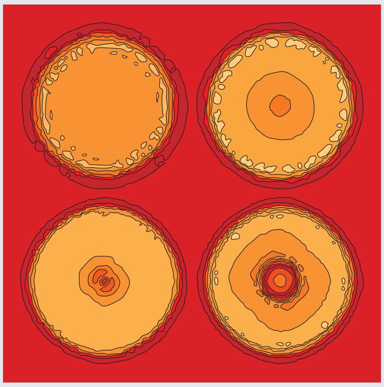
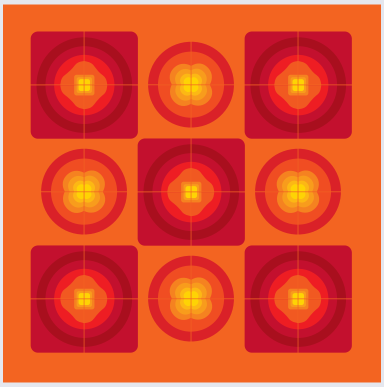
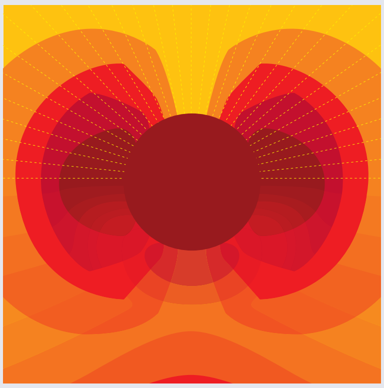




Biaxial strain on lens-shaped quantum rings of different inner radii, adapted from **Zhang *et al*** 2008 EPL **83** 67004.



Artistic impression of electrostatic particle–particle interactions in dielectrophoresis, adapted from **N Aubry and P Singh** 2006 *EPL* **74** 623.



Artistic impression of velocity and normal stress profiles around a sphere that moves through a polymer solution, adapted from **R Tuinier, J K G Dhont and T-H Fan** 2006 *EPL* **75** 929.

Image: Ornamental multiplication of space-time figures of temperature transformation rules (adapted from T. S. Bíró and P. Ván 2010 *EPL* **89** 30001; artistic impression by Frédérique Swist).

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Statistical physics of landslides: New paradigm

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Abstract – We suggest an innovative distribution function of landslide sizes, based on the non-extensive Tsallis entropy. Our result incorporates the characteristics of non-extensivity of fragmentation into the cumulative distribution of landslide sizes. Such an approach can lead to a groundbreaking statistical physics of landslides.

The distribution function of landslide sizes, in terms of areas or volumes, is fundamental for hazard risk assessment. Many "empirical" distribution functions were already proposed in the past decade, without any rigorous physical foundation. For the probability density function of landslide areas, for example, the five-parameter double Pareto distribution and the three-parameter inverse Gamma distribution were recently proposed [1,2]. A characteristic shape is commonly observed in many frequency distributions of landslide areas [2], in which the landslide frequency increases to a maximum value at the most abundant landslide size and then decreases with a power-law tail (fig. 1). Since the state-of-the-art satellite image is enough to resolve an object with the length scale of 5 meter, Malamud *et al.* therefore claimed that the decrease at the small end in probability density functions of landslide areas is real, instead of causing by incomplete data [2].

It has been argued that landslides are examples of self-organized criticality (SOC) in nature [3]. Slope instabilities develop slowly on long time scales and are relieved on short time scales. In the context of SOC, the frequency size distribution of landslides can be well described by the power-law relation. The power-law distribution is the only distribution that does not have a characteristic length scale. Power-law frequency size distributions can be explained in terms of scale invariance, *i.e.* fractal statistics. In that case, the generic rollover at the small end in the frequency distribution of landslide areas [2] cannot be expected in the SOC paradigm of landslides. We thus need a new paradigm to explain the transition from large- to small-sized landslides.

Non-extensive statistics [4,5] has been becoming a challenging framework for geophysical complex phenomena. The fundamental idea is the concept of fragmentation, in which the sum of the entropies of the parts that constitute a fractioning object after the division is larger than the entropy of the whole object [6]. For example, in the context of earthquake dynamics, the non-extensive statistical model hypothesizes that the mechanism of earthquake generation is based not only on the slippage of fault planes and the relative displacement due to the breakage of the asperities, but it is also caused by the fragments filling the space between fault planes [7,8]. Using, then, the non-extensive Tsallis formalism [5], a more realistic magnitude distribution with both the power-law tail and the kneel-down at the small magnitudes was deduced, providing an excellent fit to the seismicity of several seismic regions from a regional scale [7–10] down to the scale of the single fault [11].

This excellent fit in the magnitude distribution shows that non-extensivity describes well the source of self-similarity associated with the process' increments "infinite" variance [12]. An additional source of self-similarity may come from the process' memory (long-range temporal correlations between earthquake magnitudes, *e.g.*, see [13]), which is the case for the actual earthquake catalogues as recently shown by natural time analysis [14].

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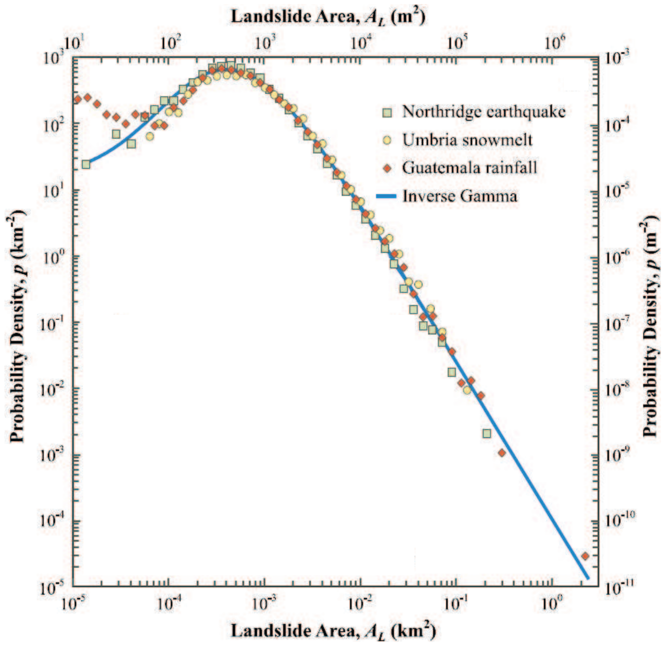

Fig. 1: (Colour on-line) Probability densities on landslide areas for three landslide datasets [2]: 11111 landslides (squares) triggered by the 1994 Northridge earthquake in California; 4233 landslides (circles) triggered by rapid snowmelt in the Umbria region of Italy in January 1997; 9594 landslides (diamonds) triggered by heavy rainfall from Hurricane Mitch in Guatemala in late October and early November 1998. The solid line is the best fit to the three datasets using the three-parameter inverse Gamma distribution suggested by Malamud *et al.* Note that the decreases at the small end in probability density functions of landslide areas are real, instead of causing by incomplete landslide data [2].

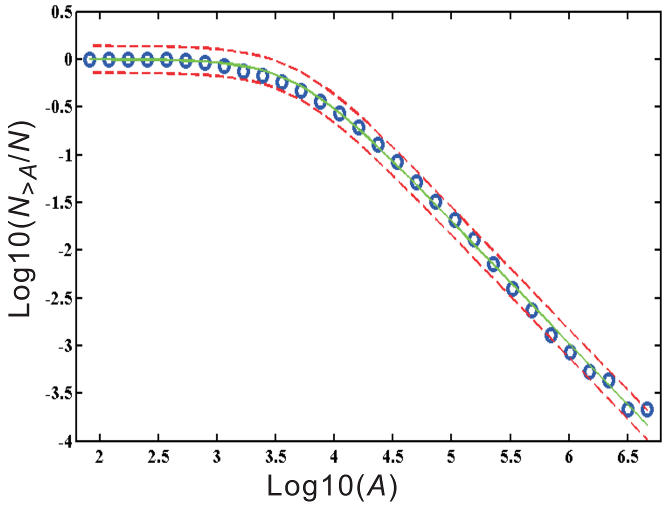

Fig. 2: (Colour on-line) Non-extensive fit (green line) of the cumulative distribution function (blue circles) of areas A (in m^2) of Taiwanese landslides induced by the Chi-Chi earthquake. The non-extensive parameters are $q = 1.539(\pm 0.007)$ and $a = 1.398(\pm 0.280) * 10^{-5}$. Two bounded red lines indicate the 95% confidence interval of observations.

Landslide is basically the fragmentation of rocks. The use of non-extensive statistics for fragments seems a straightforward tool to describe the distribution function of landslide sizes, *e.g.*, masses, volumes or areas. The Tsallis entropy for our problem is given by

$$S_q = k\frac{1-\int p^q(V)\mathrm{d}V}{q-1}, \tag{1}$$

where k is the Boltzmann's constant, $p(V)$ is the probability of finding a landslide with volume V and q is a real number. After extremizing the entropy functional S_q [6,7], the landslide size distribution function can be obtained by

$$p(V)\mathrm{d}V = \frac{(2-q)^{\frac{1}{2-q}}\mathrm{d}V}{[1+(q-1)(2-q)^{\frac{q-1}{2-q}}V]^{\frac{1}{q-1}}}. \tag{2}$$

Also, for the landslide size, the area A is another measure easy to obtain and quite often to use [2]. It is thus convenient to formulate the problem in terms of a landslide size distribution function with the function of the landslide area by means of $V \sim A^{3/2}$ [2]. The resulting expression for the area distribution function of the landslides is

$$p(A)\mathrm{d}A = \frac{C_1 A^{1/2}\mathrm{d}A}{[1+C_2A^{3/2}]^{\frac{1}{q-1}}}. \tag{3}$$

The probability of the area $p(A)$ is equal to $n(A)/N$, where $n(A)$ is the number of landslides with area A and N the total number of landslides. C_1 and C_2 are constants involving q and the proportionality constant between A and V. Therefore, based on the first principle, the maximum-entropy principle of utilizing the non-extensive Tsallis entropy, we have obtained an analytic expression for the area distribution of landslides.

To use the common frequency area distribution, the cumulative distribution function for landslide areas is, then, calculated as the integral of eq. (3) from A to infinity and given by

$$\log(N_{>A}) = \log N + \frac{2-q}{1-q}\log\left[1+a(q-1)(2-q)^{\frac{1-q}{q-2}}A^{3/2}\right], \tag{4}$$

where $N_{>A}$ is the number of landslides with areas larger than A. q and a are the non-extensive parameters.

We here demonstrate the feasibility of the non-extensive statistics applied to the area distribution of the Taiwanese landslides induced by the 1999 Chi-Chi earthquake (fig. 2). The distribution shows exactly the same features as other distributions of landslide regions mentioned in [2]. Equation (4) can fit the dataset very well with a $q \sim 1.5$. The q-value is a quantitative measure of the length scale of the spatial interactions. A q close to 1 indicates short-ranged spatial correlations. As q increases the physical state (in the sense of statistical physics) becomes much more unstable. Note that eq. (4) is not an empirical guess for the landslide size distribution but derived from the first principle of maximum non-extensive Tsallis entropy formalism, which is completely universal and has an almost unlimited range of application. The physical meaning underlying the non-extensive entropy formalism is that the final physical state can be considered as a collection of fragmented parts ν_i which, after division, have the sum of individual entropies larger than the entropy of the initial state (the union of fragmented parts). That is $\sum_i S(\nu_i) > S(\bigcup \nu_i)$. This in the context of the landslide process could be associated with the fact that the *loose-measure* volume of Earth after landslide is more than the *bank-measure* volume of Earth before landslide. When Earth is transported during landslide, it increases in volume because of an increase in voids. Also, an extra benefit in the non-extensive description of landslide sizes is that only two parameters appeared in the cumulative distribution function.

In summary, the use of non-extensive statistics is a suited tool to describe the distribution function of the fragment surfaces [7]. We here extend such a tool to describe the landslide area distribution. The obtained distribution function of landslide areas is not a mathematically trivial form. Yet, such a non-trivial result can incorporate the characteristics of non-extensivity of fragmentation into the cumulative distribution of landslide areas and explains the observed kneel-down behavior at the small landslide areas, thus fitting the real data very well. In other words, the generic *rollover* in the frequency distribution at small landslide areas [2] can be regarded as the manifestation of the physical foundation of the maximum non-extensive Tsallis entropy.

* * *

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